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Tachyon kinks

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Abstract: We study Born-Infeld type tachyonic effective action of unstable D2-brane with a runaway potential and find rich structure of static regular solitonic solutions. There exists only periodic array of tachyon kink-antikinks in pure tachyonic theory, however, in the presence of electromagnetic fields, solutions include periodic arrays, topological tachyon kinks, half kink, and bounces. Computed tension of each kink or single unit of the periodic array has $T_1 = \sqrt{2}\pi T_2$ or that with a multiplicative factor depending on electric field. When both electric and magnetic fields are turned on, fundamental string charge density has a confined component in addition to a constant piece. These evidences imply that the obtained codimension-1 objects are likely to be interpreted as D1-brane (and D1F1) or array of D1$\bar{\text{D}}$1 (and D1F1-$\bar{\text{D}}$1F1) as was the case without the electromagnetic fields. Generalization to unstable Dp-branes is straightforward.

Keywords: Brane Dynamics in Gauge Theories, Tachyon Condensation.

Contents

1. Introduction

Physics of an unstable D-brane or a system of D$\bar{\rm D}$ is manifested through the existence of tachyonic mode. The dynamics of decay of such unstable branes is described by tachyon condensation in the effective theory [1]. An efficient language to describe the decay and creation of unstable Dp-brane is to study S(pacelike)-brane [2, 3, 4]. Specific examples are the rolling tachyon [5, 6] and its family carrying electromagnetic fields [7, 8, 9]. Since they are time-dependent but spatially homogeneous classical configurations in open string theory, they are mainly used for application to various cosmological issues including inflation, dark matter, and reheating [10]. Actual decay process, however, should involve spatial inhomogeneity [11, 12] and most of the approaches attempted dynamical formation of topological kink which is a candidate of D-brane of codimension one [13]. An obstacle to generate such topological kink is so-called caustics that kink solution meets unavoidable singularity at a finite time irrespective of initial conditions, e.g., an ordinary kink, a periodic sinusoidal array.

If we want to understand the whole dynamical decay process of an unstable Dp-brane, a relevant question at the moment is to understand viable form of states after the tachyon is condensed. They cannot be perturbative excitations like tachyons or electromagnetic waves [5, 14] since all the perturbative degrees of freedom living on the unstable Dp-brane cannot survive any more once the Dp-brane decays away. In the context of open string theory, they should include D$(p-1)$-branes, fundamental strings (F1's), and their hybrids with various codimensions [15]–[21] and small fluctuations on them [7, 9].

In case of D$(p-1)$-brane, it is described by the topological kink [17]–[21] in the effective theory of tachyon field [22, 17, 23, 24, 25, 26]. Recent observation on this kind of kink [21] is noteworthy. Static topological kink in the Born-Infeld type tachyon action with a runaway potential is singular but has finite energy and tension. The world-volume theory of massless modes on this kink is again the Born-Infeld type action without any higher derivative corrections. Inclusion of fermions to this world-volume action leads to restoration of supersymmetry and κ-symmetry so that the obtained kink is identified by a BPS D$(p-1)$-brane.

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In this paper we will address an indispensable question related with the aforementioned kink [21]: can we obtain a static kink solution without singularity, which reproduces properly its singular limit? In the pure tachyonic theory of Born-Infeld type effective action with a runaway potential, the unique static regular solution is periodic array of tachyon kink-anitkinks. In the limit of vanishing pressure, it goes to that of singular topological tachyon kink-antikinks. Once Born-Infeld electromagnetism is turned on, the spectrum of regular solutions becomes rich. In addition to periodic array, there exist single topological tachyon kinks, tachyon half-kink connecting stable and unstable vacua, and tachyon bounces specified by the values of electromagnetic fields and pressure orthogonal to the soliton direction. Without or with electric field less than critical value orthogonal to the kink direction, tension of a topological kink or single unit of the periodic array has expected value, $T_1 = \sqrt{2}\pi T_2$ for superstring theory. When the electric field orthogonal to the kink direction is larger than the critical value or its transverse component is turned on, there is an additional multiplicative factor given as a function of the electric field. In the presence of the longitudinal component of the electric field, F1 charge density has a confined component and its functional form is exactly the same as those of energy density and transverse component of pressure. Proper singular limit is always taken since the energy density of them is given by a δ-function or sum of δ-functions in the singular limit of those objects. Particularly, for the regular topological tachyon kink with critical value of electromagnetic fields, all the nice analytic properties claimed in approximate form only for the singular tachyon kink [21] are saturated without any approximation. The obtained periodic array of kink-anitkinks and topological tachyon kink can be interpreted as candidates of array of D1$\bar{\text{D}}$1 (D1F1-$\bar{\text{D}}$1F1) and D1-brane (D1F1), however interpretation of half-kink and tachyon bounces are not clear, yet. Direct string computation is also lacked at the present stage.

We have now many static regular solutions. Although some of them may presumably be unstable, it is still worthwhile to study the precise nature of these configurations. Another intriguing question is on the small fluctuations on stable objects, i.e., the study of the worldvolume action of zero modes and possible existence of supersymmetry. Our approach is based on effective field theory so the obtained results should be understood in terms of string (field) theory [27, 28, 29]. Finally it would be quite interesting to investigate the role of our solutions in understanding dynamical process of D-brane decays. More realistic picture in this direction should also contain closed string degrees, e.g, gravitational field and tower of massive closed string modes [30]–[34].

The rest of the paper is organized as follows. In section 2, we consider Born-Infeld type action of a tachyon with $1/\cosh(T/T_0)$ potential and show that a periodic array of kink-antikinks is the unique static regular solution. In section 3, we turn on electric field orthogonal to the kink and find a regular topological kink in addition to a periodic array. The tension of the kink of codimension one is computed. In section 4, general form of electromagnetic fields are added to unstable D2-brane case. The obtained configurations constitute D1F1 in the form of a periodic array, kinks, half-kink, and bounces. Confinement of string charge density along the kink is achieved. We conclude with a summary of the obtained results in section 5.

2. Array of tachyon kink

In this section we consider static solutions [17]–[21] in pure tachyon model described by the following Born-Infeld type effective action [22, 17, 5, 24]

$$S = -T_p \int d^{p+1}x \; V(T)\sqrt{-\det(\eta_{\mu\nu} + \partial_\mu T \partial_\nu T)}\,, \tag{2.1}$$

where T_p is the tension of the Dp-brane. Here we use a runaway tachyon potential

$$V(T) = \frac{1}{\cosh(T/T_0)}\,, \tag{2.2}$$

which is derived from open string theory [24] and was originally introduced in ref. [9, 35, 33, 34]. In the context of string field theory, form of the action and the potential is different from our choice [23], which is also allowed due to scheme dependence. T_0 in the tachyon potential has 2 for the bosonic string and $\sqrt{2}$ for the non-BPS D-brane in the superstring.

To obtain static extended objects of codimension one, we assume that the tachyon field depends only on $x = x_1$ coordinate such as $T = T(x)$. Then, the system is governed by the only nontrivial x-component of energy-momentum conservation

$$T'_{11} (\equiv \partial_1 T_{11}) = 0\,, \tag{2.3}$$

where nonvanishing components are

$$T_{11} = -T_p \frac{V(T)}{\sqrt{1+T'^2}} < 0\,, \tag{2.4}$$

$$T_{ab} = -T_p V(T)\sqrt{1+T'^2}\,\eta_{ab}\,, \quad (\mathrm{diag}(\eta_{ab}) = (-1,1,1,\ldots,1),\ a,b = 0,2,3,\ldots,p)\,. \tag{2.5}$$

Since eq. (2.3) forces constant negative pressure $p_1 = T_{11}$ along x-direction, we can summarize our system as

$$-\frac{1}{2} = \frac{1}{2}T'^2 - \frac{1}{2}\left[\frac{T_p V(T)}{-T_{11}}\right]^2. \tag{2.6}$$

Suppose that we identify eq. (2.6) as conservation of mechanical energy $\mathcal{E}$ of a hypothetical newtonian particle in one-dimensional motion. Then this hypothetical particle has mechanical energy $\mathcal{E} = -1/2$, unit mass $m = 1$, position T at time x, and is influenced by conservative force from potential $U(T) = -(T_p V/T_{11})^2/2$. Therefore possible motions are classified by the value of $-T_{11}/T_p$ which changes shape (particularly minimum value) of the potential $U(T)$: (i) When $-T_{11}/T_p > 1$, no motion is allowed (see solid curve of $U(T)$ in figure 1). (ii) When $-T_{11}/T_p = 1$, the hypothetical particle stops at $T = 0$ eternally (see dashed curve of $U(T)$ in figure 1). (iii) When $-T_{11}/T_p < 1$, it oscillates between $T_+ = T_0 \,\mathrm{arccosh}(T_p/T_{11})$ and $T_- = -T_0 \,\mathrm{arccosh}(T_p/T_{11})$ (see dotted curve of $U(T)$ in figure 1). (iv) In the limit of $-T_{11}/T_p \to 0^+$ with keeping all other quantities, $T_\pm$ approaches positive or negative infinity, respectively.

The obtained configurations are interpreted as follows:

(ii) When the negative pressure $-T_{11}$ reaches a critical value T_p, tachyon configuration has a constant value $T = 0$ at the unstable vacuum (see dashed line in figure 2).

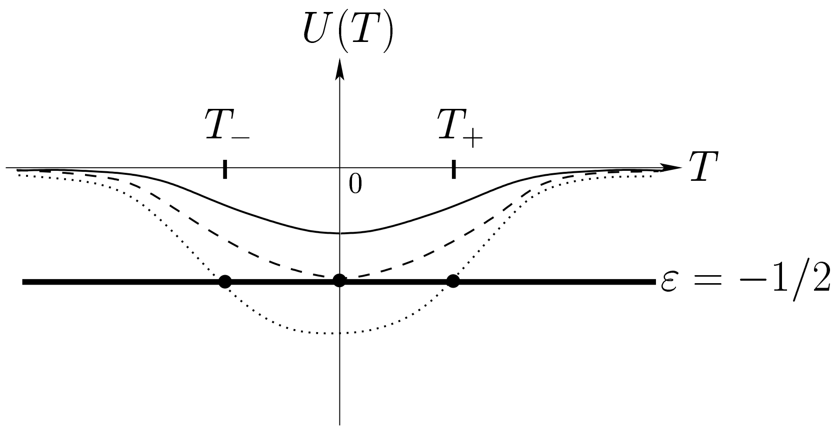

Figure 1: Shapes of $U(T) = -\frac{T_p^2}{2T_{11}^2}\mathrm{sech}^2(\frac{T}{T_0})$ for various values of $-T_{11}/T_p$.

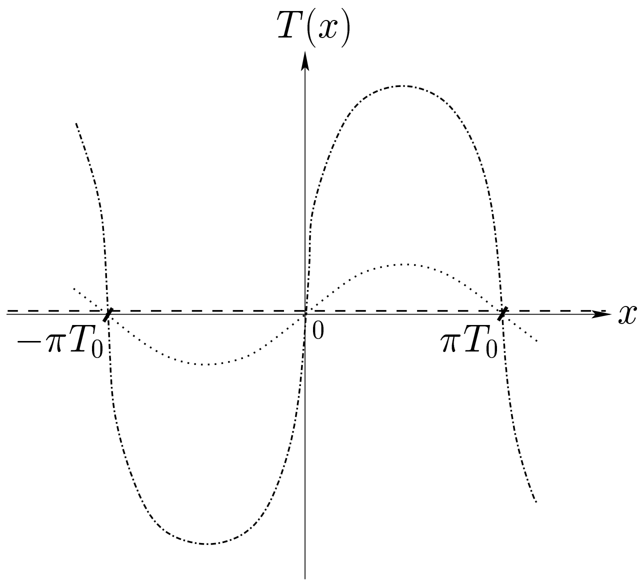

Figure 2: Profiles of tachyon field $T(x)$ for various $-T_{11}/T_p$.

(iii) When the negative pressure $-T_{11}$ is smaller than the critical value T_p, spatial inhomogeneity is turned on along the x-direction in a form of a kink, which is expressed by

$$x = \pm \int_0^T \frac{dT}{\sqrt{(\frac{T_p}{T_{11}})^2 \mathrm{sech}^2(T/T_0) - 1}}, \tag{2.7}$$

which gives

$$T(x) = T_0 \,\mathrm{arcsinh}\left[\sqrt{\left(-\frac{T_p}{T_{11}}\right)^2 - 1}\ \sin\left(\frac{x}{T_0}\right)\right]. \tag{2.8}$$

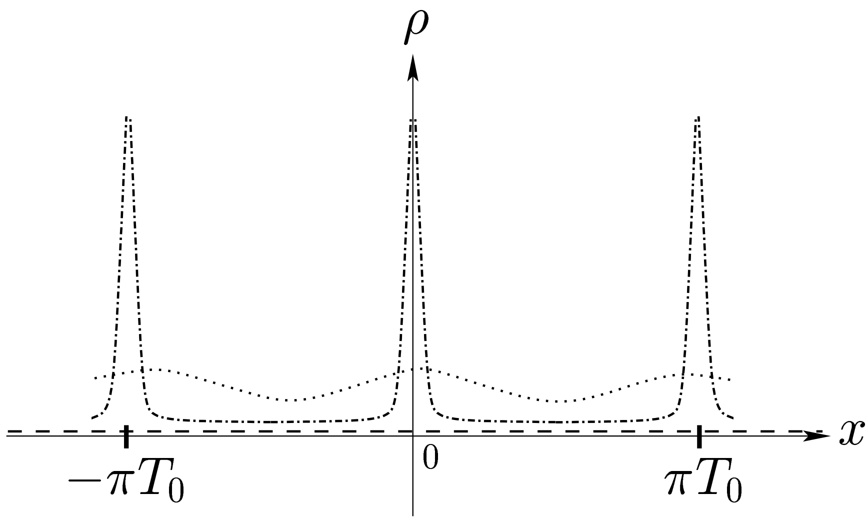

Figure 3: Profiles of energy density $\rho(x)$ and $-T_{22} = -T_{33} = \cdots = -T_{pp}$.

The corresponding energy density is given by

$$\begin{aligned}\rho \equiv T_{00} &= T_p V(T)\sqrt{1+T'^2} \\ &= \frac{-T_p^2/T_{11}}{1+\left[(\frac{T_p}{T_{11}})^2-1\right]\sin^2(x/T_0)}\,. \end{aligned} \tag{2.9}$$

The solution shows oscillating behavior between T_+ and T_- with period $2\pi T_0$ which is independent of T_{11} (see dotted curves in figure 2 and 3). Though it may be presumably unstable under a small perturbation, it is still remarkable that static nonsingular inhomogeneous solution does exist in the pure tachyon theory.

One may attempt to identify this solution as an array of kinks and antikinks. Then by integrating the energy density over a half period of the solution, we find the energy of a kink,

$$T_{p-1} \equiv \int_{-\frac{\pi}{2}T_0}^{\frac{\pi}{2}T_0} dx\, T_{00} = \pi T_0 T_p\,, \tag{2.10}$$

which can be obtained either by using the expression (2.9) directly or by replacing T' in ρ with the help of eq. (2.6),

$$T_{p-1} = -\frac{T_p^2}{T_{11}}\int_{T_-}^{T_+} dT \frac{V^2}{\sqrt{(-T_pV/T_{11})^2-1}}\,. \tag{2.11}$$

Note that eq. (2.10) is independent of the values of $T_\pm$ with the form of the potential $V(T) = 1/\cosh(T/T_0)$. In fact, it is nothing but the tension of the BPS kink identified as $D(p-1)$-brane [21]. Therefore this solution may be interpreted as representing an array of $D(p-1)\,\bar{D}(p-1)$'s.

(iv) In the limit of vanishing pressure $-T_{11}/T_p \to 0^+$, the period of the static tachyon configuration remains to be the same constant $2\pi T_0$ but profile of $T(x)$ in eq. (2.8) changes abruptly at kink or antikink sites. Accordingly, the energy density $\rho(x)$ and all other pressure components orthogonal to x-direction are more sharply localized. They have a peak value at site of the kink, $T_{00}(0) = -T_{22}(0) = -T_{33}(0) = \cdots = -T_{pp}(0) = -T_p/T_{11}$, and decreases to zero exponentially (see dashed, dotted, dotted-dashed lines in figure 3 which correspond to the cases (ii), (iii), and (iv), respectively). Eventually, the solution forms an array of step functions

$$T(x) \simeq T_0 \operatorname{arcsinh}\left[-\frac{T_p}{T_0 T_{11}} \sin\left(\frac{x}{T_0}\right)\right] \tag{2.12}$$

with an infinite gap

$$T_+ - T_- \simeq \lim_{-T_{11}/T_p \to 0} 2T_0 \ln\left(-\frac{T_p}{T_0 T_{11}}\right) \to \infty\,. \tag{2.13}$$

The energy density (2.9) then becomes

$$\rho(x) \simeq \pi T_0 T_p \sum_{n=-\infty}^{\infty} \delta(x - n\pi T_0)\,. \tag{2.14}$$

In this limit, the kinks become topological [17, 18, 19, 20, 21], and develops a singularity of step function with infinite gap at each site of kink or antikink. Note also that the formula of the tension (2.11) can be approximated as

$$T_{p-1} \simeq T_p \int_{-\infty}^{\infty} dT\, V(T)\,, \tag{2.15}$$

which coincides with that obtained in ref. [21] for a single tachyon kink with singularity.

In this section we showed that the only static regular configuration is a periodic array configuration of kink-antikinks. In its singular limit, each kink (or antikink) becomes topological, of which energy density is given by a δ-function, however its tension T_{p-1} remains to be a constant. In the previous approaches [13], time dependent kink configurations have been taken into account mostly by using initial configurations like ordinary kink or a periodic sinusoidal array for both implication to cosmological perturbation or obtaining rolling tachyons with inhomogeneity. Those solutions seem to be suffered by encountering of unavoidable singularity at a finite time so-called caustics. Probably it is intriguing to study the instability of aforementioned array configuration (2.8) under a small perturbation during time evolution where $-T_{11}$ is an adjustable parameter for preparation of an initial configuration.

3. Regular kink with electric field

In this section, we demonstrate that there exists a static, regular, tachyon kink solution when the electric field is larger than or equal to the critical value.

As the simplest case, here we will only examine the case of unstable D2-brane of which Born-Infeld type action of a tachyon T and an abelian gauge field A_μ is

$$S = -T_2 \int d^3x\, V(T) \sqrt{-\det(\eta_{\mu\nu} + \partial_\mu T \partial_\nu T + F_{\mu\nu})}\,, \tag{3.1}$$

where T_2 is tension confined on the D2-brane. The generalization to higher dimensions should be straightforward.

Let us introduce a few notations $\bar{\eta}_{\mu\nu} = \eta_{\mu\nu} + \partial_\mu T \partial_\nu T$, $\bar{\eta} \equiv \det(\bar{\eta}_{\mu\nu})$, and $\bar{F}_{\mu\nu} = F_{\mu\nu}$. Define $X_{\mu\nu} \equiv \bar{\eta}_{\mu\nu} + \bar{F}_{\mu\nu}$ and $X \equiv \det(X_{\mu\nu})$. From the definition, X is simplified to

$$X = \bar{\eta}\left(1 + \frac{1}{2}\bar{F}_{\mu\nu}\bar{F}^{\mu\nu}\right), \tag{3.2}$$

where the transformation rule of a contravariant barred field strength tensor $\bar{F}^{\mu\nu}$ is

$$\bar{F}^{\mu\nu} = \bar{\eta}^{\mu\alpha}\bar{\eta}^{\nu\beta}F_{\alpha\beta}\,, \qquad \bar{\eta}^{\mu\nu} = \eta^{\mu\nu} - \frac{\partial^\mu T \partial^\nu T}{1 + \partial_\rho T \partial^\rho T}\,. \tag{3.3}$$

Equations of motion derived from the action (3.1) are

$$\partial_\mu\left(\frac{V}{\sqrt{-X}}C_{\rm S}^{\mu\nu}\partial_\nu T\right) + \sqrt{-X}\frac{dV}{dT} = 0\,, \tag{3.4}$$

$$\partial_\mu\left(\frac{V}{\sqrt{-X}}C_{\rm A}^{\mu\nu}\right) = 0\,. \tag{3.5}$$

Here $C_{\rm S}^{\mu\nu}$ and $C_{\rm A}^{\mu\nu}$ are symmetric and antisymmetric parts of the cofactor, respectively

$$C^{\mu\nu} = \bar{\eta}(\bar{\eta}^{\mu\nu} - \bar{F}^{*\mu}\bar{F}^{*\nu} + \bar{F}^{\mu\nu}) = C_{\rm S}^{\mu\nu} + C_{\rm A}^{\mu\nu}\,, \tag{3.6}$$

where barred dual field strength tensor of 1-form $\bar{F}^{*\mu}$ is

$$\bar{F}^{*\mu} = \frac{\bar{\epsilon}^{\mu\nu\rho}}{2}\bar{F}_{\nu\rho} \qquad \left(\bar{\epsilon}^{\mu\nu\rho} = \frac{\epsilon^{\mu\nu\rho}}{\sqrt{-\bar{\eta}}}, \epsilon^{012} = -1\right). \tag{3.7}$$

Energy-momentum tensor is

$$T_{\mu\nu} = -T_2\frac{V(T)}{\sqrt{-X}}\left[-\eta_{\mu\nu}X + \frac{1}{2}\left(C_{\mu\rho}(\partial_\nu T\partial^\rho T + F_\nu{}^\rho) + C_{\nu\rho}(\partial_\mu T\partial^\rho T + F_\mu{}^\rho)\right)\right], \tag{3.8}$$

where $C_{\mu\nu} = \eta_{\mu\alpha}\eta_{\nu\beta}C^{\alpha\beta}$.

Suppose all fields are static. Since our final goal is to obtain a straight kink configuration, we assume that the tachyon field depends only on the x-direction $T = T(x)$. In this section, for simplicity, we will also consider only the case $\mathbf{E} = E(\mathbf{x})\hat{\mathbf{x}}$ and $B = 0$. General case will be considered in the next section. Then Bianchi identity $\partial^\mu F_\mu^* = 0$ which is nothing but three-dimensional analogue of Faraday's law

$$\frac{\partial B}{\partial t} = \epsilon_{0ij}\partial_i E_j, \qquad \left(E_i = F_{0i}, B = \frac{\epsilon_{0ij}F_{ij}}{2}\right) \tag{3.9}$$

implies $E = E(x)$. The equations for the gauge field (3.5) result in constancy of conjugate momentum Π

$$\Pi' = 0\,, \tag{3.10}$$

where Π and X (3.2) are

$$\frac{\Pi}{E} = T_2\frac{V}{\sqrt{-X}} \geq 0\,, \tag{3.11}$$

$$-X = 1 - E^2 + T'^2 \geq 0\,. \tag{3.12}$$

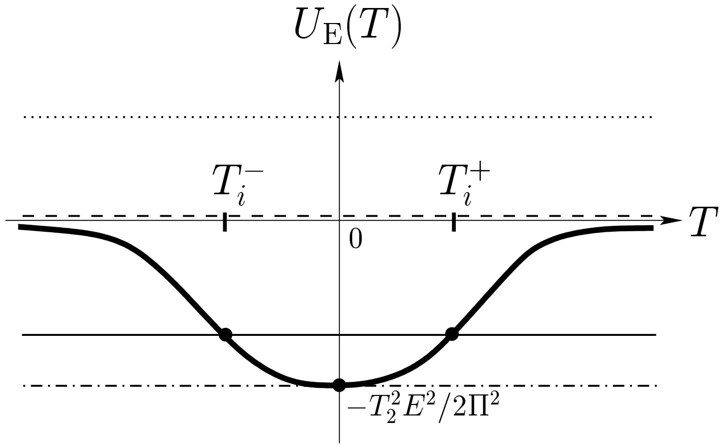

Figure 4: Shape of $U_{\rm E}(T)$.

Conservation of the energy-momentum tensor $\partial_\mu T^{\mu\nu}=0$ reduces to constant pressure along x-direction, $-T'_{11}=(\Pi/E)'=0$, so that the electric field E itself is a constant. It means that the solutions are classified by two independent parameters, (Π,E) or equivalently $(-T_{11},E)$. In the context of string theory, the existence of the constant background electric field E and its conjugate momentum density Π is interpreted as that of a string fluid consisting of straight F1's along x-axis [7, 8, 9]. Here we assume positive E, Π, and $-T_{11}$ for convenience without loss of generality. Therefore, from eqs. (3.11)–(3.12), we obtain a first-order differential equation for T, consistent with the tachyon equation (3.4),

$$\mathcal{E}_{\rm E}=\frac{1}{2}T'^2+U_{\rm E}(T)\,, \tag{3.13}$$

where

$$\mathcal{E}_{\rm E}=-\frac{1}{2}(1-E^2)\,, \tag{3.14}$$

$$U_{\rm E}(T)=-\frac{T_2^2E^2}{2\Pi^2}V(T)^2=-\frac{T_2^2E^2}{2\Pi^2}\frac{1}{\cosh^2(T/T_0)}\,. \tag{3.15}$$

(See figure 4 for a schematic shape of $U(T)$.) Note that for static configurations the energy density (3.8) becomes

$$\rho\equiv T_{00}=T_2\frac{V}{\sqrt{-X}}(1+T'^2)=\Pi E+\frac{E}{\Pi}\left[T_2V(T)\right]^2, \tag{3.16}$$

and pressure T_{22} orthogonal to the configuration is

$$p_2\equiv T_{22}=-\frac{\Pi}{E}(1-E^2+T'^2)=-\frac{E}{\Pi}[T_2V(T)]^2\le 0\,. \tag{3.17}$$

Non-constant piece of the energy density (3.16) coincides with y-component of the pressure (3.17) with opposite signature. Constant piece of the energy density (3.16) is proportional to Π but the second term given by square of the tachyon potential is inversely proportional to Π so that constant piece dominates in large Π limit and vice versa in small Π limit. For the latter, the pressure orthogonal to the configuration also becomes large.

Tachyon configurations determined by eqs. (3.13)–(3.15) are classified by the value of $\mathcal{E}_{\rm E}$. When $\mathcal{E}_{\rm E} < U(0)$, ($E^2 > 1/[1 + (T_2/\Pi)^2]$), no real tachyon configuration is allowed. When $\mathcal{E} = U(0)$, (i.e., $E^2 = 1/[1 + (T_2/\Pi)^2]$; see dotted-dashed line in figure 4), we have the ontop solution $T(x) = 0$ with the corresponding energy density $\rho = \Pi E\left[1 + (T_2/\Pi)^2\right]$ (the dotted-dashed lines in figures 5 and 6). For $U_{\rm E}(0) < \mathcal{E} < 0$, ($1/[1 + T_2^2/\Pi^2] < E^2 < 1$; solid line in figure 4), we obtain spatially periodic inhomogeneous configuration similar to that (2.7) in the previous section,

$$x = \int_0^T \frac{dT}{\sqrt{E^2[1 + T_2^2/\Pi^2 \cosh^2(T/T_0)] - 1}}\,, \tag{3.18}$$

which gives

$$T(x) = T_0 \,\mathrm{arcsinh}\left[\sqrt{\frac{E^2T_2^2}{\Pi^2(1-E^2)} - 1}\ \sin\left(\frac{\sqrt{1-E^2}}{T_0}x\right)\right]. \tag{3.19}$$

The tachyon field oscillates between the "turning points" in figure 4, $T_i^{\pm} = \pm T_0\,\mathrm{arccosh}\left(\frac{T_2E}{\Pi\sqrt{1-E^2}}\right)$ with period $2\pi\zeta = 2\pi T_0/\sqrt{1-E^2}$. This solution is the generalization of the array of kinks found in section 2 in the presence of electric field in the transverse direction of kinks. Comparing eq. (3.19) with $E = 0$ case (2.8), we find that turning on the electric field increases the period and also the effective tension $T_2 \to T_2/\sqrt{1-E^2}$. This kind of phenomenon was expected from the beginning through a rescaling of x-coordinate in the effective action (3.1)

$$\begin{aligned} S &= -T_2 \int dt\,dx\,dy\,V(T)\sqrt{1 - E^2 + \left(\frac{dT}{dx}\right)^2} \\ &= -T_2 \int dt\,d(\sqrt{1-E^2}x)\,dy\,V(T)\sqrt{1 + \left[\frac{dT}{d(\sqrt{1-E^2}x)}\right]^2}\,. \end{aligned} \tag{3.20}$$

Substituting the solution (3.19) into eq. (3.16), we have

$$\rho - \Pi E = -p_2 = \frac{E}{\Pi}\frac{T_2^2}{1 + \left[\frac{E^2T_2^2}{\Pi^2(1-E^2)} - 1\right]\sin^2(x/\zeta)}\,. \tag{3.21}$$

The tachyon profile $T = T(x)$ and the energy density $\rho = \rho(x)$ for this periodic solution are represented as the solid lines in figure 5 and 6.

As in section 2 we integrate the energy density of the kink, i.e. the localized piece of $\rho(x)$, over the half period to get its tension ($p = 2$ in the present case)

$$\begin{aligned} T_{p-1} &= \frac{ET_2^2}{\Pi}\int_{-\frac{\pi}{2}\zeta}^{\frac{\pi}{2}\zeta} dx\,V^2(T(x)) \\ &= \pi T_0 T_p\,. \end{aligned} \tag{3.22}$$

It is exactly the same value as the tension T_{p-1} without electric field (2.10) and again independent of the "turning points" $T_i^{\pm}$. Moreover it is independent of the electric field E

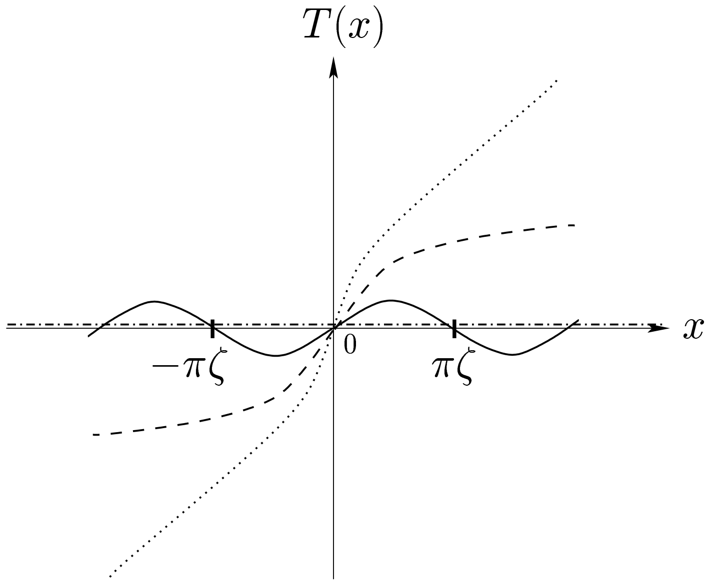

Figure 5: Profiles of tachyon field $T(x)$.

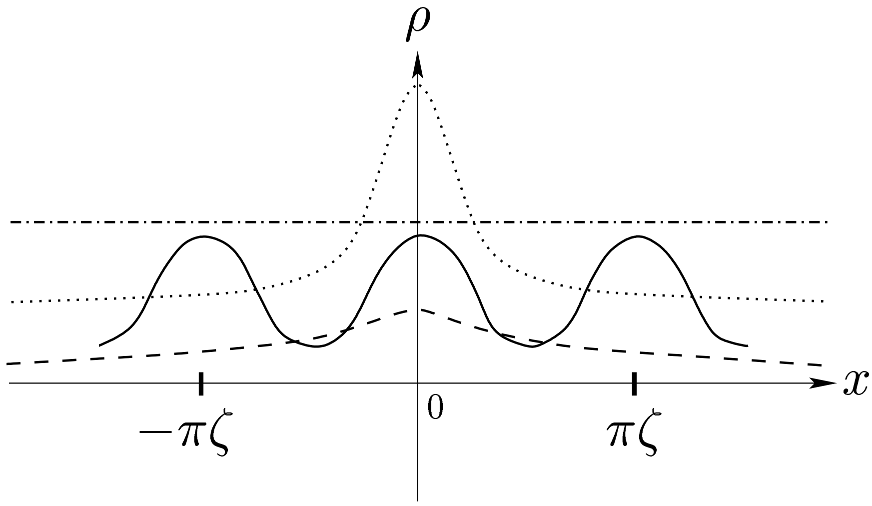

Figure 6: Profiles of energy density $\rho(x)$.

or F1 charge density Π in the transverse direction. In the limit of $\Pi \to 0$ with fixed E, which corresponds to $-T_{11} \to 0$, both the energy density (3.16) and the pressure (3.17) are given by sums of δ-functions

$$\rho(x) \simeq T_{p-1} \sum_{n=-\infty}^{\infty} \delta(x - n\pi T_0/\sqrt{1-E^2}) \simeq -p_2 \,. \tag{3.23}$$

As $\mathcal{E}$ approaches 0^-, i.e., when E^2 approaches 1 (the dashed line in figure 4), $T_i^\pm$ stretches to infinity and we obtain new types of solutions, which do not exist in the limit of vanishing electric field. In fact the solution becomes a regular static single-kink configuration with $T'(\pm\infty) = 0$ (the dashed line in figure 5),

$$T(x) = T_0 \,\mathrm{arcsinh}\left(\frac{T_2}{\Pi T_0}x\right). \tag{3.24}$$

This regularity can also be understood through localization of the action (3.1) as follows

$$\begin{aligned} S &= -T_p \int dt\, dx\, d^{p-1}y\, V(T)\sqrt{1 - E^2 + T'^2} \\ &\overset{E=\pm 1}{=} -(\pm)T_p \int dt\, dx\, d^{p-1}y\, V(T)T' && (3.25) \\ &= -\int dt\, d^{p-1}y\, T_p \int_{-\infty}^{\infty} dT\, V(T)\,, && (3.26) \end{aligned}$$

where $+$ $(-)$ in the second line (3.25) corresponds to the kink (the antikink). The exact integral formula for the tachyon field in the third line (3.26) is nothing but that of the tension T_{p-1} (2.15) obtained through rather complicated manipulation only for the singular kink [21]. In this case, the energy density ρ is given in a particularly simple form,

$$\rho - \Pi = -p_2 = \pi T_0 T_2 \cdot \frac{\xi/\pi}{x^2 + \xi^2}\,, \tag{3.27}$$

where $\xi \equiv \Pi T_0/T_2$ represents the width of the kink. Note that as Π goes to zero the localized energy density approaches a δ-function with energy $T_{p-1} = \pi T_0 T_p$, while large Π broadens the width.

Finally, when $\mathcal{E}_{\mathrm{E}} > 0$, ($E^2 > 1$; see the dotted line in figure 4), the solution is given by

$$T(x) = T_0 \,\mathrm{arcsinh}\left[\sqrt{1 + \frac{E^2 T_2^2}{\Pi^2(E^2-1)}}\,\sinh\left(\frac{\sqrt{E^2-1}}{T_0}x\right)\right], \tag{3.28}$$

which has a finite asymptotic slope $T'(\pm\infty) \neq 0$. The energy density (3.16) for this solution also has a constant and a localized piece which coincides with the pressure p_2 with opposite signature

$$\rho(x) - \Pi E = -p_2 = \frac{ET_2^2}{\Pi}\frac{1}{1 + \left[1 + \frac{E^2T_2^2}{\Pi^2(E^2-1)}\right]\sinh^2\left(\frac{\sqrt{E^2-1}}{T_0}x\right)}. \tag{3.29}$$

Similar to the previous case, we obtain

$$\begin{aligned} T_{p-1} &= \frac{ET_p^2}{\Pi}\int_{-\infty}^{\infty} dx V^2(T(x)) \\ &= 2T_0T_p \arctan\left(\frac{ET_2}{\Pi\sqrt{E^2-1}}\right) && (3.30) \end{aligned}$$

which is no longer a constant and is less than $\pi T_0 T_p$. Of course, $E \to 1^+$ limit with fixed Π reproduces trivially the previous case of topological kink (3.24) and thereby its tension T_{p-1} approaches $\pi T_0 T_p$. The profiles of $T(x)$ and the energy density $\rho(x)$ are plotted as dotted lines in figure 5 and figure 6. As $\mathcal{E} \to 0^+$, the slope at spatial infinity $T'(\pm\infty)$ goes to zero as expected. For huge electric field limit $E \to \infty$, the tachyon kink becomes sharply peaked, $T'(\pm\infty) \to \pm\infty$, with keeping finite tension.

All the array and kinks are specified by two independent constants, x-component of electric field E and conjugate momentum Π for a given D2-brane. In synthesis for fixed Π, as the electric field E increases, irregularity is induced in both energy density and transverse component of the pressure and becomes sharply peaked, and its pattern changes from an array to single kink.

4. Confined F1 charge along the tachyon kink

In this section we consider the most general configuration of the static electromagnetic fields with x-dependence alone

$$\mathbf{E} = E_1(x)\hat{\mathbf{x}} + E_2(x)\hat{\mathbf{y}}, \qquad B = B(x)\,. \tag{4.1}$$

Since the system of our interest is still unstable D2-brane, we can use the formulas in eqs. (3.1)–(3.8). If we insert eq. (4.1) into the Faraday's law (3.9), it forces constancy of y-component of the electric field E_2. Conjugate momenta Π_i of the gauge fields are

$$\Pi_1 = T_2 \frac{V(T)}{\sqrt{-X}} E_1\,, \tag{4.2}$$

$$\Pi_2 = T_2 \frac{V(T)}{\sqrt{-X}} (1 + T'^2) E_2\,. \tag{4.3}$$

From time-component of the gauge equations (3.5), we see that Π_1 should be a constant, while Π_2 need not be. Note that Π_2 is nonzero only when the parallel component of the electric field E_2 is turned on. Now the y-component of the energy-momentum conservation $\partial_1 T^{12} = 0$ is automatically satisfied and the x-component, $\partial_1 T^{11} = 0$, gives

$$\frac{\Pi_1}{E_1} = T_2 \frac{V(T)}{\sqrt{-X}} = \text{a positive constant}, \tag{4.4}$$

so that x-component of the electric field E_1 is also a constant. The y-component of the gauge equations (3.5) dictates constancy of the magnetic field B. Therefore, all the electromagnetic fields $(\mathbf{E}, B)$ are actually independent of x. Then the remaining time-component of the energy-momentum conservation is also satisfied automatically: $\partial_1[(T_2 V/\sqrt{-X})(E_2 + E_1 B)B] = 0$.

Eliminating non-constant Π_2 in the remaining two equations (4.2)–(4.3), we can again summarize dynamics of our system by a single first-order equation as done in the previous sections

$$\mathcal{E}_{\rm EM} = \frac{1}{2} T'^2 + U_{\rm EM}(T)\,, \tag{4.5}$$

where

$$\mathcal{E}_{\rm EM} = -\frac{1 - \mathbf{E}^2 + B^2}{2(1 - E_2^2)}\,, \tag{4.6}$$

$$U_{\rm EM}(T) = -\frac{T_2^2 E_1^2}{2\Pi_1^2 (1 - E_2^2)} V(T)^2 = -\frac{T_2^2 E_1^2}{2\Pi_1^2 (1 - E_2^2)} \frac{1}{\cosh^2(T/T_0)}\,. \tag{4.7}$$

In the limit of $E_2 \to 0$ and $B \to 0$, it is consistent with eqs. (3.13)–(3.15). Therefore, all the solutions are classified by a set of four parameters, i.e., (Π_1, E_1, E_2, B) or (T_{11}, E_1, E_2, B) where

$$-T_{11} = \frac{\Pi_1}{E_1}(1 - E_2^2)\,, \tag{4.8}$$

which has sign flip at the critical value of $E_2 = \pm 1$.

In this setup, some components of the energy-momentum tensor are nonvanishing constants,

$$\begin{aligned} T_{0i} &= -\frac{\Pi_1}{E_1}\epsilon_{0ij}E_j B\,, \\ T_{12} &= -\frac{\Pi_1}{E_1}E_1 E_2\,. \end{aligned} \tag{4.9}$$

The other components of the the energy-momentum tensor have nontrivial x-dependence:

$$\begin{aligned} \rho \equiv T_{00} &= T_2 \frac{V}{\sqrt{-X}}(1 + T'^2 + B^2) \\ &= \frac{\Pi_1(E_1^2 - B^2 E_2^2)}{E_1(1 - E_2^2)} + \frac{E_1}{\Pi_1(1 - E_2^2)}[T_2 V(T)]^2 \\ &\equiv \rho_{\rm c} + \rho_{\rm l}\,, \end{aligned} \tag{4.10}$$

$$\begin{aligned} p_2 \equiv T_{22} &= -T_2 \frac{V}{\sqrt{-X}}(1 + T'^2 - E_1^2) \\ &= -\frac{\Pi_1(E_1^2 E_2^2 - B^2)}{E_1(1 - E_2^2)} - \frac{E_1}{\Pi_1(1 - E_2^2)}[T_2 V(T)]^2 \\ &\equiv p_{2\rm c} + p_{2\rm l}\,. \end{aligned} \tag{4.11}$$

Note also that the string charge density Π_2 in the y-direction has essentially the same x-dependence,

$$\begin{aligned} \Pi_2 &= \frac{\Pi_1 E_2(E_1^2 - B^2)}{E_1(1 - E_2^2)} + \frac{E_1 E_2}{\Pi_1(1 - E_2^2)}[T_2 V(T)]^2 \\ &\equiv \Pi_{2\rm c} + \Pi_{2\rm l}\,. \end{aligned} \tag{4.12}$$

These three inhomogeneous quantities, namely energy density (4.10), pressure (4.11), and string charge density (4.12) in y-direction share a few properties which may be related to confinement of D1F1: (i) They are composed of a constant and a common x-dependent part, $\Pi_{2\rm l}(x) = E_2\rho_{\rm l}(x) = -E_2 p_{2\rm l}(x)$. (ii) The constant term is proportional to Π_1 but the localized piece is inversely proportional to Π_1. (iii) They have an overall multiplicative factor $1/(1 - E_2^2)$ so that they flip signature at the critical value $E_2 = \pm 1$.

Now let us study solution spectra of eqs. (4.5)–(4.7) in what follows. First, note that the coefficient of the potential term in eq. (4.7) changes the sign at $E_2^2 = 1$. Therefore we will separately examine the system according to the value of E_2 (we assume $E_2 \geq 0$ with no loss of generality): $0 \leq E_2 < 1$, $E_2 = 1$ and $E_2 > 1$.

When E_2 is smaller than one, there is not much to do since eq. (4.5) is essentially identical to that in the previous section. With some trivial replacements of parameters, we obtain the following result. (See figures 5 and 6 for the possible types of solutions.)

(i) When $E_2 < 1$ and $1 - \mathbf{E}^2 + B^2 > T_2^2 E_1^2/\Pi_1^2$, no solution exists.

(ii) When $E_2 < 1$ and $1 - \mathbf{E}^2 + B^2 = T_2^2 E_1^2/\Pi_1^2$, there is only a constant solution $T(x) = 0$.

(iii) When $E_2 < 1$ and $0 < 1 - \mathbf{E}^2 + B^2 < T_2^2 E_1^2/\Pi_1^2$, we have the solution of kink-antikink array,

$$T(x) = T_0 \,\text{arcsinh}\left[\sqrt{u^2-1}\sin\left(\frac{x}{\zeta_B}\right)\right], \tag{4.13}$$

where u and ζ_B are defined by

$$u^2 = \frac{E_1^2 T_2^2}{\Pi_1^2 |1 - \mathbf{E}^2 + B^2|}, \tag{4.14}$$

$$\zeta_B = \frac{T_0}{\sqrt{2|\mathcal{E}_{\rm EM}|}} = \sqrt{\left|\frac{1-E_2^2}{1-\mathbf{E}^2+B^2}\right|}\, T_0 \,. \tag{4.15}$$

Then the square of the tachyon potential $V(T)^2$, to which the localized pieces of ρ, p_2 and Π_2 are proportional, is given by

$$V^2(T(x)) = \frac{1}{1 + (u^2-1)\sin^2(x/\zeta_B)} \,. \tag{4.16}$$

From integration of the localized part of the energy density $\rho(x)$ along x-axis, we obtain the tension T_1 of codimension one object

$$T_1 = \frac{\pi T_0 T_2}{\sqrt{1-E_2^2}}\,, \tag{4.17}$$

which is larger than the previous case (3.22) by a multiplicative factor $1/\sqrt{1-E_2^2}$. This is expected since the energy density of a Born-Infeld theory increases precisely by this factor when a constant electric field is turned on on the world volume.

In the limit of $E_2 \to 1$, T_1 diverges and oscillation becomes rapid $\zeta_B \to 0$ with infinite peak of the energy density. Let us take another limit $\Pi_1 \to 0$ which leads to $u \to \infty$ but ζ_B remains finite. Then $V(T)^2$ is written by a sum of δ-functions so that we have

$$\rho(x) \simeq \frac{\pi T_0 T_2}{\sqrt{1-E_2^2}} \sum_{n=-\infty}^{\infty} \delta(x - 2\pi\zeta_B)\,, \tag{4.18}$$

$$\Pi_2 \simeq \frac{\pi T_0 T_2 E_2}{\sqrt{1-E_2^2}} \sum_{n=-\infty}^{\infty} \delta(x - 2\pi\zeta_B)\,. \tag{4.19}$$

We can read the tension of each kink (or antikink) $\pi T_0 T_2/\sqrt{1-E_2^2}$ from eq. (4.18) and string charge density of each kink $\pi T_0 T_2 E_2/\sqrt{1-E_2^2}$ from eq. (4.19).

(iv) When $1-\mathbf{E}^2+B^2 \to 0$, both u and ζ_B diverge with finite ratio $\zeta_B/u = \frac{\Pi_1 T_0}{E_1 T_2}\sqrt{1-E_2^2}$. Since $V(T)^2$ takes a lorentzian shape

$$V^2(T) \to \pi\frac{\zeta_B}{u}\frac{\zeta_B/\pi u}{x^2 + (\zeta_B/u)^2}\,, \tag{4.20}$$

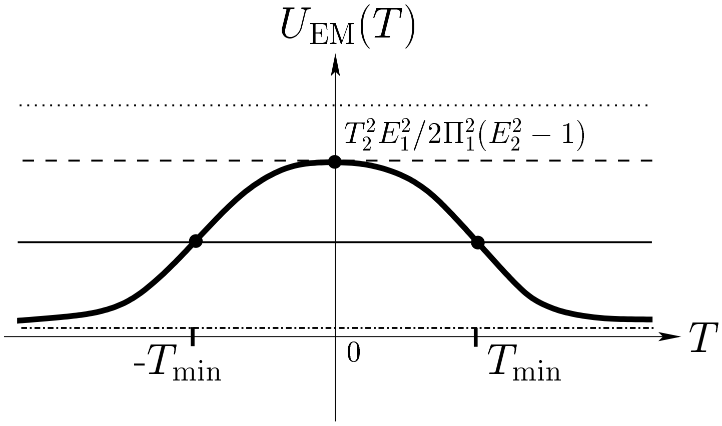

Figure 7: Shape of $U_{\rm EM}(T)$ when $|E_2| > 1$.

so do the localized pieces of energy density (4.10), transverse pressure (4.11), and string charge density (4.12). This case corresponds to a single-kink solution as in the previous section. The configuration of tachyon kink is given by

$$T(x) = T_0 \,\mathrm{arcsinh}\left(\frac{ux}{\zeta_B}\right). \tag{4.21}$$

In this limit, the action (3.1) is rewritten again in a localized form

$$\begin{aligned} S &= -T_p \int dt\,dx\,dy\, V(T)\sqrt{1-\mathbf{E}^2+B^2+(1-E_2^2)T'^2} \\ &\overset{1-\mathbf{E}^2+B^2=0}{=} -(\pm)\sqrt{1-E_2^2}\;T_p\int dt\,dx\,dy\,V(T)T' \qquad (4.22)\\ &= -\int dt\,dy\sqrt{1-E_2^2}\;T_p\int_{-\infty}^{\infty} dT\,V(T)\,, \qquad (4.23)\end{aligned}$$

where + (−) in the second line (4.22) corresponds to the kink (the antikink). In the third line (4.23) we obtain the same formula of tension (2.15) with the aforementioned overall factor $\sqrt{1-E_2^2}$.

(v) Finally, when $E_2 < 1$ and $1-\mathbf{E}^2 = B^2 < 0$, we obtain the solution similar to eq. (3.28). The details are omitted.

Now we consider the case $E_2 = 1$. In this case the solution is trivial since T' disappears, for example, in eq. (4.5) or in the tachyon equation of motion (3.4). Then the only remaining equation is $dV/dT = 0$. Therefore there are only homogeneous solutions $T = 0$ or $T = \pm\infty$.

As mentioned before, once the magnitude of E_2 is larger than 1, the property of our system changes drastically because of the sign flip of the potential $U_{\rm EM}(T)$ as shown in figure 7. Similar to the analysis of the previous solutions, tachyon configurations are classified by the value of $\mathcal{E}_{\rm EM}$ (4.6). In fact, the solution configurations should be analogous to rolling tachyon solutions in unstable D2-brane with electromagnetic fields when $1-\mathbf{E}^2+$

$B^2 \geq 0$. From eq. (3.1) the action of our system is rewritten as

$$\begin{aligned} S &= -T_2 \int dt\, dx\, dy\, V(T) \sqrt{1 - \mathbf{E}^2 + B^2 - (E_2^2 - 1)\left(\frac{dT}{dx}\right)^2} \\ &= -T_2 \sqrt{E_2^2 - 1} \int dt\, d\left(\frac{T_0 x}{\zeta_B}\right) dy V(T) \sqrt{1 - \left[\frac{dT}{d(T_0 x/\zeta_B)}\right]^2} . \end{aligned} \tag{4.24}$$

Note that the signature of x-direction flips when $E_2 > 1$. This action is actually exactly the same as that of rolling tachyon which is given by

$$\begin{aligned} S &= -T_2 \int dt\, dx\, dy\, V(T) \sqrt{1 - \mathbf{E}^2 + B^2 - (1 + B^2)\left(\frac{dT}{dt}\right)^2} \\ &= -T_2 \sqrt{1 + B^2} \int d\left(\frac{T_0 t}{\xi}\right) dx\, dy V(T) \sqrt{1 - \left[\frac{dT}{d(T_0 t/\xi)}\right]^2} , \end{aligned} \tag{4.25}$$

where

$$\xi = \sqrt{\frac{1 + B^2}{1 - \mathbf{E}^2 + B^2}} T_0 . \tag{4.26}$$

Thus, there exists a one-to-one correspondence between a kink solution with spatial distribution and the time evolution of a homogeneous rolling tachyon solution. With this identification, the pressure $-T_{11}$, in our system corresponds to the hamiltonian density $\mathcal{H}$, in the rolling tachyon system.

Now we describe the solutions in detail when $E_2 > 1$.

(i) As $\mathcal{E}_{\rm EM} \to 0^+$, (i.e, $\mathbf{E}^2 - B^2 \to 1$; see the dot-dashed lines in figures 7–9), static constant vacuum solution is allowed at $T(x) = \pm\infty$. Since the tachyon potential vanishes for $T = \pm\infty$, the localized terms in $\rho(x)$, $p_2(x)$, and $\Pi_2(x)$ are all zero.

(ii) When the energy $\mathcal{E}_{\rm EM}$ is larger than zero but smaller than the top of tachyon potential (i.e., $1 - (T_2 E_1/\Pi_1)^2 < \mathbf{E}^2 - B^2 < 1$; see the solid line in figure 7), there is a turning point $T_{\rm min}$ such that

$$|T|(x) \geq T_{\rm min} = T_0 \,\mathrm{arccosh}(u) . \tag{4.27}$$

As shown by the solid curves in the figure 8, configuration is convex up (or convex down) so we will call this solution a tachyon bounce. Explicit form of the solution is given by

$$T(x) = T_0 \,\mathrm{arcsinh}\left[\sqrt{u^2 - 1} \cosh\left(\frac{x}{\zeta_B}\right)\right] , \tag{4.28}$$

where u and ζ_B are given in eqs. (4.14) and (4.15), respectively. Its asymptotic slopes are

$$T'(\pm\infty) = \pm \frac{T_0}{\zeta_B} , \tag{4.29}$$

which are shown by the two solid straight lines in figure 8. Note that, since $E_2 > 1$, the localized parts of $\rho(x)$, $p_2(x)$, and $\Pi_2(x)$ all flip their signs. For example, the energy density

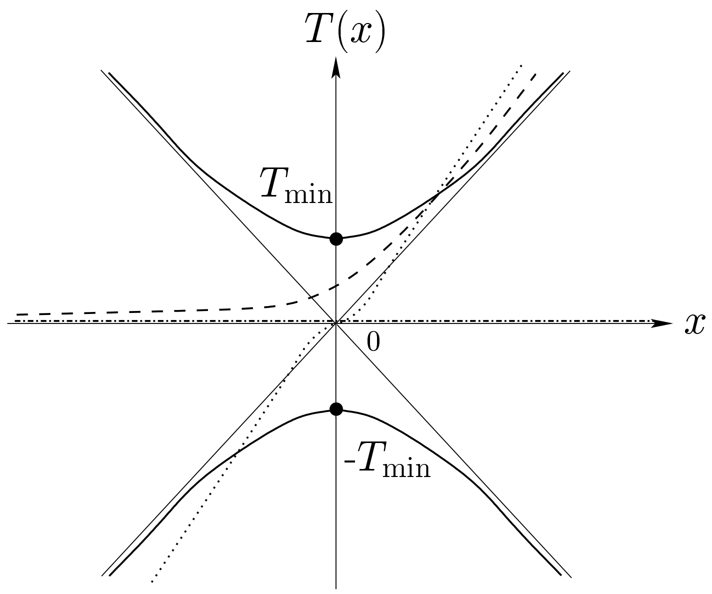

Figure 8: Profiles of tachyon kink and bounce $T(x)$.

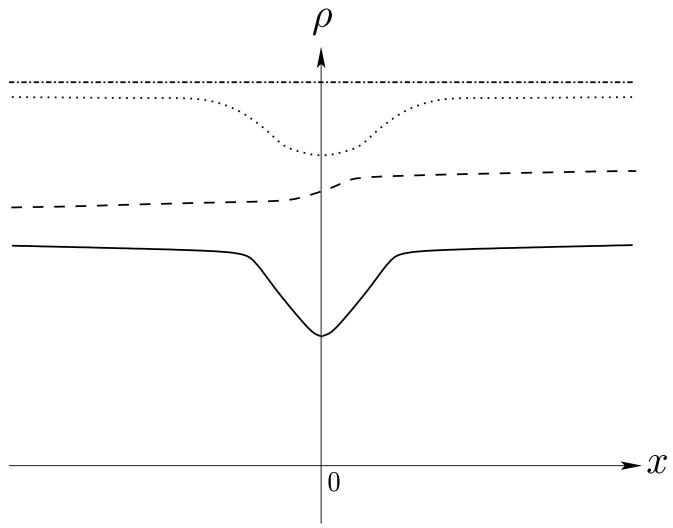

Figure 9: Profiles of energy density $\rho(x)$.

has positive constant term and a negative localized contribution near the origin (see the solid line in figure 9)

$$\rho = \frac{\Pi_1(B^2E_2^2 - E_1^2)}{E_1(E_2^2-1)} - \frac{E_1T_2^2}{\Pi_1(E_2^2-1)}\frac{1}{1+(u^2-1)\cosh^2(x/\zeta_B)}\,. \tag{4.30}$$

Also it means that the transverse component of pressure p_2 is positive and the string charge density Π_2 is negative.

(iii) When the "energy" $\mathcal{E}_{\rm EM}$ is the same as the maximum of the "potential" $U_{\rm EM}(0)$, ($\mathbf{E}^2 - B^2 = 1 - (T_2E_1/\Pi_1)^2$; see the dashed line in figure 7), we obtain the trivial vacuum ontop solution $T(x) = 0$. In addition, there is also the tachyon "half-kink" (or "half-antikink") solution which connects the unstable symmetric vacuum $T(-\infty) = 0$ and a stable broken vacuum $T(\infty) = \pm\infty$ (see the dashed curve in figure 8),

$$T(x) = \pm T_0 \,\text{arcsinh}\left[\exp\left(\frac{x}{\zeta_B}\right)\right]. \tag{4.31}$$

Since the half-kink connects two vacua with different vacuum energy as $V(T=0) > V(T = \pm\infty)$, the energy density is monotonically increasing (see the dashed curve in figure 9),

$$\rho(x) = \frac{\Pi_1(B^2E_2^2 - E_1^2)}{E_1(E_2^2-1)} - \frac{E_1T_2^2}{\Pi_1(E_2^2-1)}\frac{1}{1+\exp(2x/\zeta_B)}. \tag{4.32}$$

The transverse component of pressure p_2 and the string charge density Π_2 are also monotonic.

(iv) If $\mathcal{E}_{\rm EM} > U_{EM}(0)$, ($\mathbf{E}^2 - B^2 < 1 - (T_2E_1/\Pi_1)^2$, see the dotted line in figure 7), we have

$$T(x) = T_0 \,\text{arcsinh}\left[\sqrt{1-u^2}\sinh\left(\frac{x}{\zeta_B}\right)\right]. \tag{4.33}$$

Configuration is monotonically increasing (or decreasing) from $T(-\infty) = \mp\infty$ to $T(\infty) = \pm\infty$ (see also the dotted curve in figure 8). Opposite to the similar kink solutions in the previous section, slope of the solutions has minimum value at the origin, and maximum at infinity. This solution can be considered as two half-kink solutions joined at the origin. The energy density of this solution is given by

$$\rho = \frac{\Pi_1(B^2E_2^2 - E_1^2)}{E_1(E_2^2-1)} - \frac{E_1T_2^2}{\Pi_1(E_2^2-1)}\frac{1}{1+(1-u^2)\sinh^2(x/\zeta_B)}. \tag{4.34}$$

It is plotted in figure 9 with the dotted line.

5. Conclusion

In this paper, static solutions have been investigated in Born-Infeld type tachyonic effective action with and without electromagnetic fields.

In pure tachyonic theory, we have shown that the periodic array of tachyon kink-antikinks is the unique static regular solution. In the limit of vanishing pressure along the array, the solution becomes an array of step functions with an infinite gap. The values of tension of single unit kink (or antikink) $T_{p-1} = \sqrt{2}\pi T_p$ and period remain constant irrespective of the value of pressure.

When the electrostatic field orthogonal to the tachyon soliton is turned on in an unstable D2-brane, a periodic array, and regular topological tachyon kinks are obtained, classified by the value of electric field with fixed conjugate momentum. For a given electric field, taking limit of vanishing conjugate momentum leads to singular limit where both energy density and transverse component of pressure are given by sum of δ-functions or a δ-function. When the electric field is smaller than or equal to critical value, tension is $T_{p-1} = \sqrt{2}\pi T_p$. When the electric field is larger than the critical value, a multiplicative factor depending on the value of electric field appears.

For the general case with both electric and magnetic fields, spectra of solutions are specified by four parameters, three components of electromagnetic fields and one pressure component along the soliton, and divided into two classes by transverse component of electric field. When it is smaller than critical value, the spectra of solitons are exactly the same as the cases of pure electric field, i.e., they are periodic array of tachyon kink-antikinks and topological tachyon kinks. Minor difference appears in the constant scales and the tension. Major difference is that they carry confined component of F1 charge density so that each kink is presumably a D1F1. When the transverse component of electric field is larger than critical value, we additionally found half-kink connecting unstable and stable vacua, tachyon bounce, and topological tachyon kink.

The topological tachyon kink at the critical value of the electric field orthogonal to the tachyon soliton seems to be a BPS object, but it should be addressed in future works by obtaining its worldvolume action and checking existing supersymmetry as has been done in ref. [21].

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